CS 70   Discrete Mathematics and Probability Theory

Summer 2021   DIS 6A

1   Max of Uniforms

Let $X_1, ... X_n$ be independent $U[0,1]$ random variables, and let $X = \max(X_1, ... X_n)$. Compute each of the following in terms of n.

(a) What is the cdf of X?

(b) What is the pdf of X?

(c) What is $\mathbb{E}[X]$?

(d) What is $Var[X]$?

2   Darts with Friends

Michelle and Alex are playing darts. Being the better player, Michelle's aim follows a uniform distribution over a disk of radius r around the center. Alex's aim follows a uniform distribution over a disk of radius $2r$ around the center.

(a) Let the distance of Michelle's throw from the center be denoted by the random variable X and let the distance of Alex's throw from the center be denoted by the random variable Y.

- What's the cumulative distribution function of X?
- What's the cumulative distribution function of Y?
- What's the probability density function of X?

- What's the probability density function of Y?

(b) What's the probability that Michelle's throw is closer to the center than Alex's throw? What's the probability that Alex's throw is closer to the center?

(c) What's the cumulative distribution function of $U = \min\{X,Y\}$?

(d) What's the cumulative distribution function of $V = \max\{X,Y\}$?

(e) What is the expectation of the absolute difference between Michelle's and Alex's distances from the center, that is, what is $\mathbb{E}[|X - Y|]$? [*Hint*: Use parts (c) and (d), together with the continuous version of the tail sum formula, which states that $\mathbb{E}[Z] = \int_0^\infty P(Z \geq z)dz$.]