Due: Sunday 6/27, 10:00 PM
Grace period until Sunday 6/27, 11:59 PM

Sundry

Before you start writing your final homework submission, state briefly how you worked on it. Who else did you work with? List names and email addresses. (In case of homework party, you can just describe the group.)

1 Administrivia

(a) Make sure you are on the course Piazza (for Q&A) and Gradescope (for submitting homeworks, including this one). Find and familiarize yourself with the course website. What is its homepage's URL?

(b) Read the policies page on the course website.

(i) What is the breakdown of how your grade is calculated?
(ii) What is the attendance policy for small discussions? Big discussions?
(iii) When are vitamins released and when are they due?
(iv) How many vitamins do you have to complete to get full credit?

2 Course Policies

Go to the course website and read the course policies carefully. Leave a followup in the Homework 0, Question 2 thread on Piazza if you have any questions. Are the following situations violations of course policy? Write "Yes" or "No", and a short explanation for each.

(a) Alice and Bob work on a problem in a study group. They write up a solution together and submit it, noting on their submissions that they wrote up their homework answers together.

(b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.

(c) Erin comes across a proof that is part of a homework problem while studying course material. She reads it and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.

(d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.

(e) Heidi has completed her homework using LaTeX. Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution, and Irene uses it to write her own solution with a citation to Heidi.

(f) Joe found homework solutions before they were officially released, and every time he got stuck, he looked at the solutions for a hint. He then cited the solutions as part of his submission.

3 LaTeX

If you have ample time on your hands and would like to learn a new skill, you may be interested in learning LaTeX! LaTeX is a document preparation system that puts mathematical formulae into nicely formatted documents. Using LaTeX can help you organize your thought process and make lives easier for readers. We have provided some resources on the course website to help you get started with using LaTeX. Feel free to ask questions on Piazza if you have any questions.

For this question, try to typeset the following formulas. This will give you some practice writing mathematical formulas properly. **If you choose to hand-write and scan your solutions, just write out the formulas by hand.**

Hint: You may find the *amsmath* LaTeX package helpful.

(a) $\forall x \exists y \left((P(x) \land Q(x,y)) \implies x \leq \sqrt{y}\right)$

(b)

$$\sum_{i=0}^{k} i = \frac{k(k+1)}{2}$$

4 Logical Equivalence?

Decide whether each of the following logical equivalences is correct and justify your answer.

(a) $\forall x \left(P(x) \land Q(x)\right) \equiv \forall x\, P(x) \land \forall x\, Q(x)$

(b) $\forall x \left(P(x) \lor Q(x)\right) \equiv \forall x\, P(x) \lor \forall x\, Q(x)$

(c) $\exists x \left(P(x) \lor Q(x)\right) \equiv \exists x\, P(x) \lor \exists x\, Q(x)$

(d) $\exists x \left(P(x) \land Q(x)\right) \equiv \exists x\, P(x) \land \exists x\, Q(x)$

5 Propositional Practice

In parts (a)-(b), convert the English sentences into propositional logic. In parts (c) - (d), convert the propositions into English. For parts (b) and (d), use the notation $a \mid b$ to denote the statement "a divides b", and use the notation $P(x)$ to denote the statement "x is a prime number".

(a) For every real number k, there is a unique real solution to $x^3 = k$.

(b) If p is a prime number, then for any two natural numbers a and b, if p doesn't divide a and p divides ab, then p divides b.

(c) $(\forall x, y \in \mathbb{R})\,[(xy = 0) \implies ((x = 0) \vee (y = 0))]$

(d) $\neg((\exists y \in \mathbb{N})\,[(\forall x \in \mathbb{N})\,[(x > y) \implies ((y \mid x) \vee P(x))]])$

6 Prove or Disprove

For each of the following statements, (1) decide whether or not the statement is false, and (2) provide a rigorous proof backing up your answer.

(a) For all natural numbers n, $n^2 + 3n$ is even if and only if n is odd.

(b) For all real numbers r, if r is irrational then $r + 1$ is irrational.

7 Proof Checker

Your friend has sent you three proofs, listed below. As a CS 70 student, your job is to determine whether each proof is correct or incorrect. If the proof is correct, then just write "the proof is correct". If the proof is incorrect, clearly and precisely describe what the logical error is. Note: We are not looking for answers such as "the claim is false"; we want you to point out the fault in the reasoning!

(a) **Claim**: for all $n \in \mathbb{N}$, $(2n+1 \text{ is a multiple of } 3) \implies (n^2+1 \text{ is a multiple of } 3)$.

Proof: We will use proof by contraposition. Assume $2n+1$ is not a multiple of 3. We have the following cases.

- If $n = 3k+1$ for $k \in \mathbb{N}$, then $n^2+1 = 9k^2+6k+2$ is not a multiple of 3.
- If $n = 3k+2$ for $k \in \mathbb{N}$, then $n^2+1 = 9k^2+12k+5$ is not a multiple of 3.
- If $n = 3k+3$ for $k \in \mathbb{N}$, then $n^2+1 = 9k^2+18k+10$ is not a multiple of 3.

In all cases, we have concluded n^2+1 is not a multiple of 3, so we have proved the claim.

(b) **Claim**: For all real numbers a, b, if $a+b \geq 20$ then $a \geq 17$ or $b \geq 3$.

Proof We will again use a proof by contraposition. Suppose that $a < 17$ and $b < 3$ (note that this is equivalent to $\neg(a \geq 17 \vee b \geq 3)$). Since $a < 17$ and $b < 3$, $a+b < 20$ (note that

$a+b<20$ is equivalent to $\neg\big(a+b\geq 20\big)$). Thus, if $a+b\geq 20$, then $a\geq 17$ or $b\geq 3$ (or both, as "or" is not "exclusive or" in this case). By contraposition, for all real numbers a,b, if $a+b\geq 20$ then $a\geq 17$ or $b\geq 3$.