(1) Goals of this class

- Learn mathematical tools useful for CS

- Learn to think like a mathematician

Today:

② An Example: Basic Arithmetic

(I promise this is relevant)

How to count to 10 in Basque

1	bat
2	bi
3	hiru
4	lau
5	bost
6	sei
7	zazpi
8	zortzi
9	bederatzi
10	hamar

Take a few seconds to look over the list

Question What is woru + lau?

Question Count backwards from hamar to bat

The point:

Optimistic: Even if things in this class seem hard or confusing, you can master them with practice

Pessimistic: You will need to put in time & effort

Just watching lecture =

③ My advice

- See all the material multiple times

- Give yourself plenty of time to think about homework problems

Monday morning: Old hw already due, no new lectures yet.

④ Propositional Logic

Onward to the actual content!

Def A proposition is

Example

$1+1=3$

Sacramento is the capital of California.

$x+x=y$

Please don't read this sentence.

Def The truth value of a proposition is true if
and false

4.1 Combining Propositions

We can form new propositions from old ones

Example

Use letters to denote propositional variables

Def If P and Q are propositions:

operation	symbols	meaning
Negation		
Conjunction		
Disjunction		

Example P = "I am a carrot" Q = "I am an avocado"

4.2 Truth Tables

A way to systematically record what an operation on propositions is doing

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	

4.3 More ways of combining propositions

Def If P and Q are propositions:

Implication
Bi-implication

P	Q	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T		
T	F		
F	T		
F	F		

Warning!!

Example If Berkeley is the capital of California, then I am a carrot.

(4.4) More about implication

Other ways to express $P \Rightarrow Q$ in English.

P = "You like CS 70"
Q = "You like probability"

If you like CS 70 then you like probability.
You like CS 70 only if you like probability.
You like probability if you like CS 70.
The fact that you like probability is a necessary condition for you to like CS 70.

<u>Def</u> Given an implication $P \Rightarrow Q$.
The converse is the implication
The contrapositive is the implication
The inverse is the implication

5 Logical Equivalence

Def A propositional formula is an expression made up of propositional variables combined with operations like $\neg, \wedge, \vee, \Rightarrow$

Example

Def A propositional formula is a tautology if

Example $P \vee \neg P$

P	$\neg P$	$P \vee \neg P$
T	F	
F	T	

$P \vee Q$

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

Goal of math:

Def Two propositional formulas are logically equivalent
if

Example $P \Rightarrow Q$ and $\neg P \vee Q$

P	Q	$\neg P$	$P \Rightarrow Q$	$\neg P \vee Q$
T	T			
T	F			
F	T			
F	F			

5.1 Some interesting examples of logical equivalence

Question Which of the following is logically equivalent to $P \Rightarrow Q$?

- Converse $(Q \Rightarrow P)$
- Contrapositive $(\neg Q \Rightarrow \neg P)$

Answer:

P	Q	$\neg Q$	$\neg P$	$P \Rightarrow Q$	$Q \Rightarrow P$	$\neg Q \Rightarrow \neg P$
T	T					
T	F					
F	T					
F	F					

Example If you like CS70, then you like probability.

More examples

$\neg(P \wedge Q) \equiv$

$\neg(P \vee Q) \equiv$

⑥ Quantifiers

We can write more interesting statements using quantifiers.

Recall $x+x=y$ is not a proposition

Def A propositional function is

Example Let $P(x,y)$ be the statement "$x+x=y$"
$P(1,2)$ is
$P(3,17)$ is

Def The domain or universe of a propositional function $P(x)$ is

Def If $P(x)$ is a propositional function then

$\forall x\, P(x)$ denotes the proposition

$\exists x\, P(x)$ denotes the proposition

Write $\forall x \in S\ P(x)$ or $\exists x \in S\ P(x)$ to indicate that

Example $\forall x \in \mathbb{N},\ x \geq 0$

$\exists x \in \mathbb{N},\ x \geq 0$

$\forall x \in \mathbb{N}\ \exists y \in \mathbb{N},\ x + x = y$

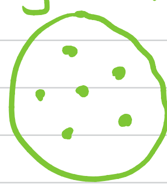
<u>Example</u> "Every nonzero rational number can be multiplied by some rational number to get 1"

"There are at least 2 integers"

6.1 Logical Equivalence with Quantifiers

Def Two propositions built up out of propositional functions, $\neg$, $\wedge$, $\vee$, $\to$ and quantifiers are logically equivalent if

Example $\exists x \exists y\, P(x,y) \equiv \exists y \exists x\, P(x,y)$

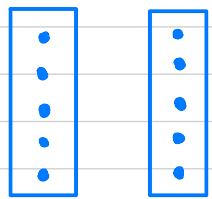


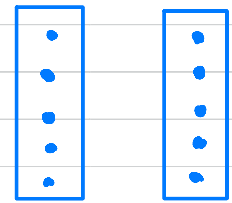
De Morgan's laws for quantifiers

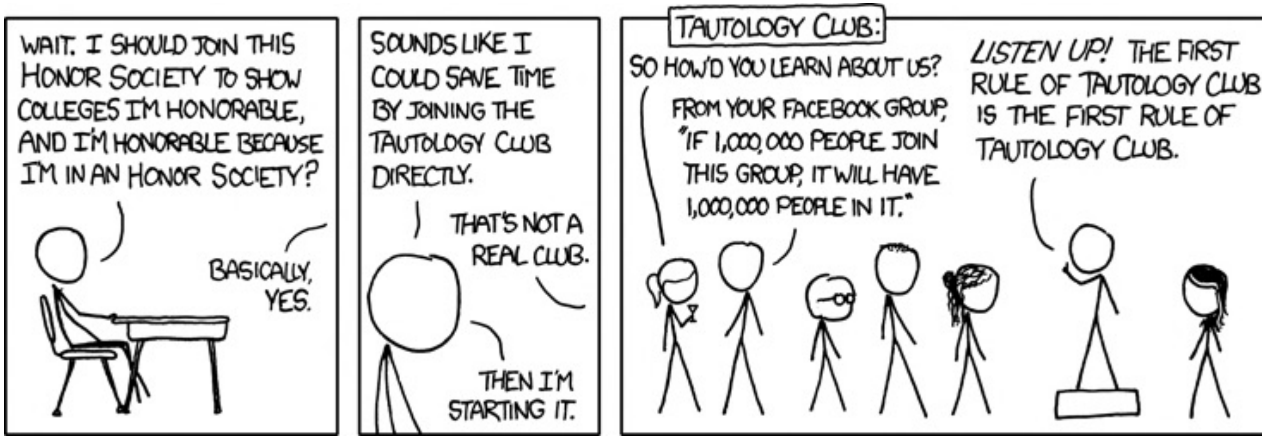
$\neg(\forall x\, P(x)) \equiv$

$\neg(\exists x\, P(x)) \equiv$

Example Are $\forall x \exists y\, P(x,y)$ and $\exists y \forall x\, P(x,y)$ logically equivalent?






WAIT. I SHOULD JOIN THIS HONOR SOCIETY TO SHOW COLLEGES I'M HONORABLE, AND I'M HONORABLE BECAUSE I'M IN AN HONOR SOCIETY?
BASICALLY, YES.
SOUNDS LIKE I COULD SAVE TIME BY JOINING THE TAUTOLOGY CLUB DIRECTLY.
THAT'S NOT A REAL CLUB.
THEN I'M STARTING IT.
TAUTOLOGY CLUB:
SO HOW'D YOU LEARN ABOUT US?
FROM YOUR FACEBOOK GROUP, "IF 1,000,000 PEOPLE JOIN THIS GROUP, IT WILL HAVE 1,000,000 PEOPLE IN IT."
LISTEN UP! THE FIRST RULE OF TAUTOLOGY CLUB IS THE FIRST RULE OF TAUTOLOGY CLUB.

xkcd #703