O Proofs Def A proof is 600d: Bad: How do you confirm your beloefs are correct? Math Science

Def A proof is ~ finite list of statements, each of which is logically implied by the previous statement, which is used to establish the truth of some proposition.

226	theorem nat_abs_add_le (a b : \mathbb{Z}) : nat_abs (a + b) \leq nat_abs a + nat_abs b :=	
227	begin	
228	have : \forall (a b : \mathbb{N}), nat_abs (sub_nat_nat a (nat.succ b)) \leq nat.succ (a + b),	
229	{ refine (λ a b : ℕ, sub_nat_nat_elim a b.succ	
230	(λ m n i, n = b.succ → nat_abs i ≤ (m + b).succ) rfl);	
231	intros i n e,	
232	{ subst e, rw [add_comm _ i, add_assoc],	
233	exact nat.le_add_right i (b.succ + b).succ },	
234	{ apply succ_le_succ,	
235	rw [← succ.inj e, ← add_assoc, add_comm],	
236	<pre>apply nat.le_add_right } },</pre>	
237	cases a; cases b with b b; simp [nat_abs, nat.succ_add];	
238	try {refl}; [skip, rw add_comm a b]; apply this	
239	end	

My advice: Imagine your proof is being read by

For now:

2 How to prove things How you should prove a proposition depends on the logical structure of the proposition Structure How to prove it PAQ PAD PQQ $\exists x \in S, P(x)$ $\forall x \in S, P(x)$ Can also replace the proposition to be proved with a logically equivalent proposition that has a different structure.

Example

(3) Direct proof
A direct proof
Thim For every natural number, there is a natural
number greater than it.
proof
Reminder

$$\forall x \in S, P(x)$$
 Let a be an arbitrary element of S
 $\exists x \in S, P(x)$ Provide some a $\in S$ and prove $P(a)$

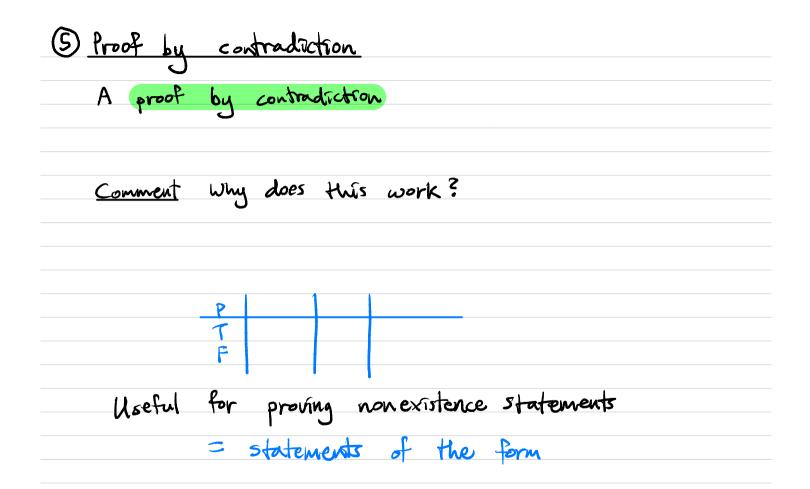
Def Given n, m E Z, we say n drivides m, voritten Example Example This For all a, b, n ∈ Z, if n | a and n | b then n | (a-b) proof Assume P 1s true and prove Q Reminder P>Q

One lesson:

(4) Proof by contraposition A proof by contraposition and odd *Pff* Fact nEZ is even iff The For every neZ, if n² is even then so is n. Example Direct proof? proof Let n be an integer.

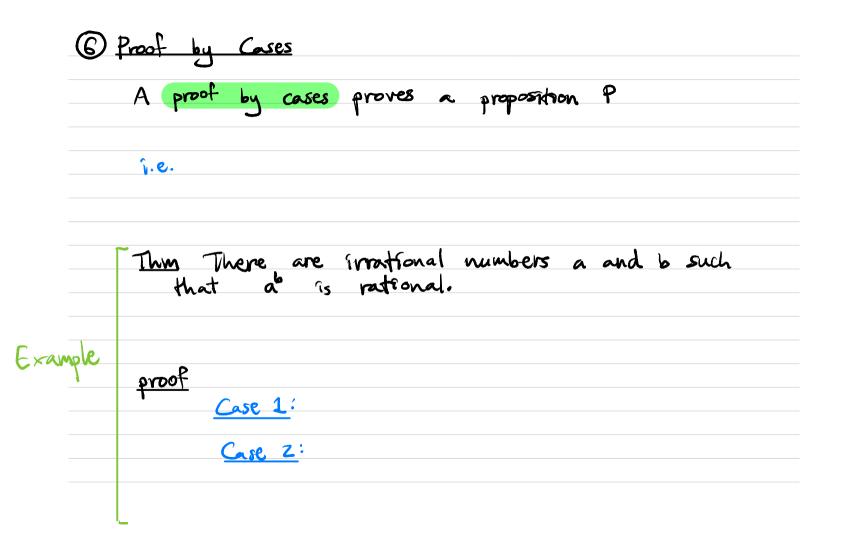
Proof by contraposition is especially useful if you are trying to prove something of the form

Def A real number r is rational 2f Otherwise, r is irrational. Thus For every real number a, if a is irrational then so is 3a. Example proof Let a be a real number.

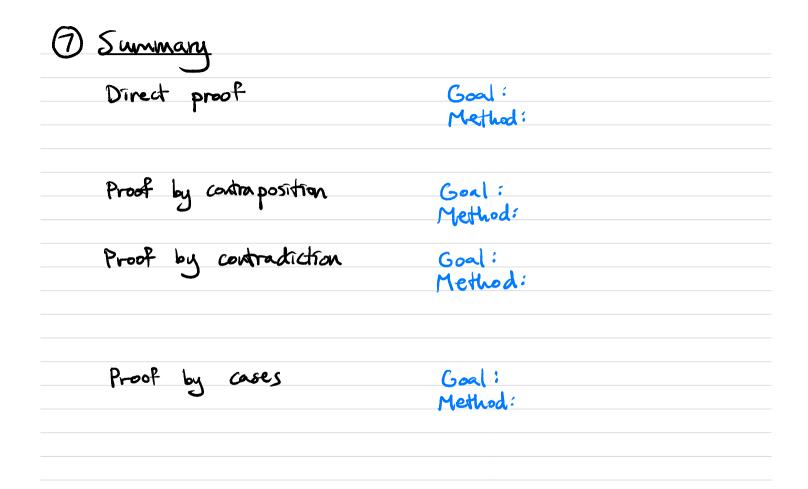


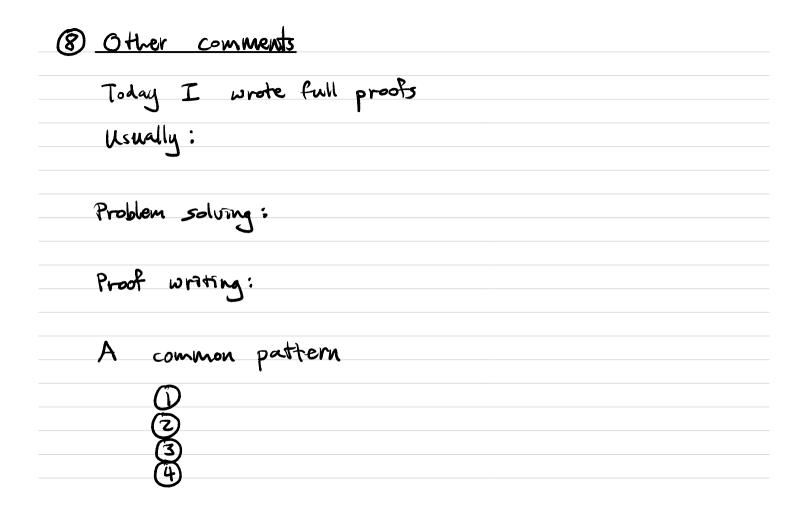
Def A natural number is prime if Example Fact Every natural number greater than 1 Example This (Euclid?) There are infinitely many prime numbers proof Suppose for contradiction that there are only finitely many primes, p,, pz, ..., pn.

Fact If $a \in \mathbb{Q}$ then there are $p, q \in \mathbb{Z}$ such that $q \neq 0$, $a = \frac{p}{q}$ and Thin JZ is irrational. proof Suppose for contradiction JZ is rational. amole



Sometimes proof by cases is really cool, other times... Fact For every natural number n, there is a natural number K such that one of the following holds: The For all nEN, 3/(n3-n) proof Let n be a natural number and Case 1: Case 2: Case 3:





Challenge question Can you find a propositional formula using only $P, Q, and \Lambda$ which is logically equivalent to $P \Rightarrow Q$? If not, can you prove it? what about logically equivalent to PAQ using only P, Q, and =>?