

- ① Joint PDFs
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Joint PDFs

Let X and Y be two continuous random variables.

Then the joint density function $f_{X,Y}: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ satisfies :

Joint CDFs

Let X and Y be two random variables.
Then the joint cumulative distribution function
 $F_{x,y} : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ satisfies:

Conditional PDFs

Let X and Y be two continuous random variables with joint density function $f_{x,y}$

For any y with $f(y) > 0$, the conditional distribution of X given $Y=y$ is defined as:

When Y is continuous, even though $P(Y=y) = 0$, if $f_y(y) > 0$,

Independence

Let X and Y be two continuous random variables.
 X and Y are independent if:

Marginalization

To recover the individual pdfs from the joint pdf:

$$f_X(x) =$$

$$f_Y(y) =$$

Example: $X, Y \sim \text{Unif}(0, 2)$

What is $f_{X,Y}(x,y)$ for two uniform r.v.s on $[0, 2]$?

Uniform Density Over a Disk : Joint

What is $f_{x,y}(x,y)$ for a uniform density over a disk of radius r centered at the origin?

Uniform Density Over a Disk: Marginals

What is $f_y(y)$ and $f_x(x)$, now that we know the joint over the disk?

Uniform Density over a Disk: Conditional PDFs.

What is $f_{x|y=y}(x)$?

What about $\text{Cov}(X, Y)$?

2D LOTUS

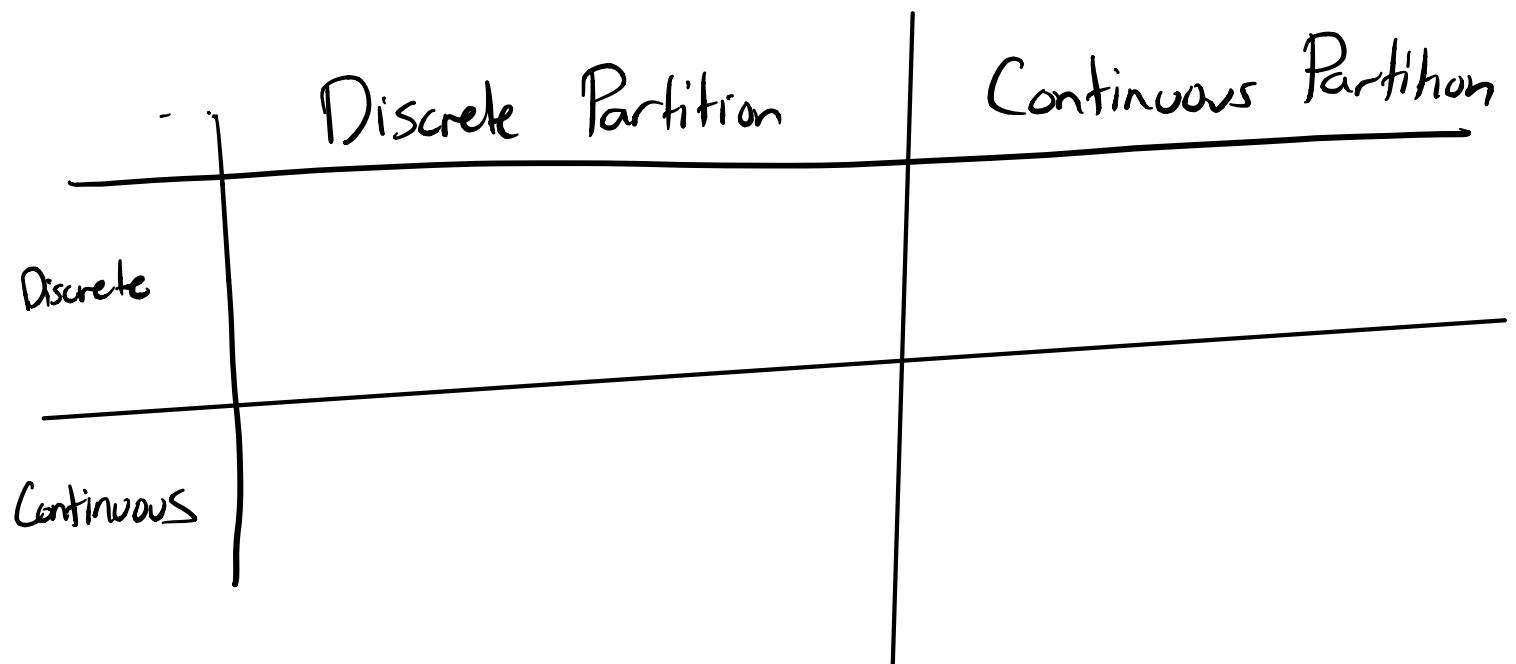
Let X, Y be two random variable with joint PDF $f_{X,Y}(x,y)$, and let $g(x,y)$ be a real-valued function of x, y . Then,

Expected Distance Between Two Points

Let $X, Y \stackrel{\text{i.i.d}}{\sim} \text{Unif}(0,1)$. What is $E[|X-Y|]$?

$$E[|X-Y|] =$$

Total Probability Theorem

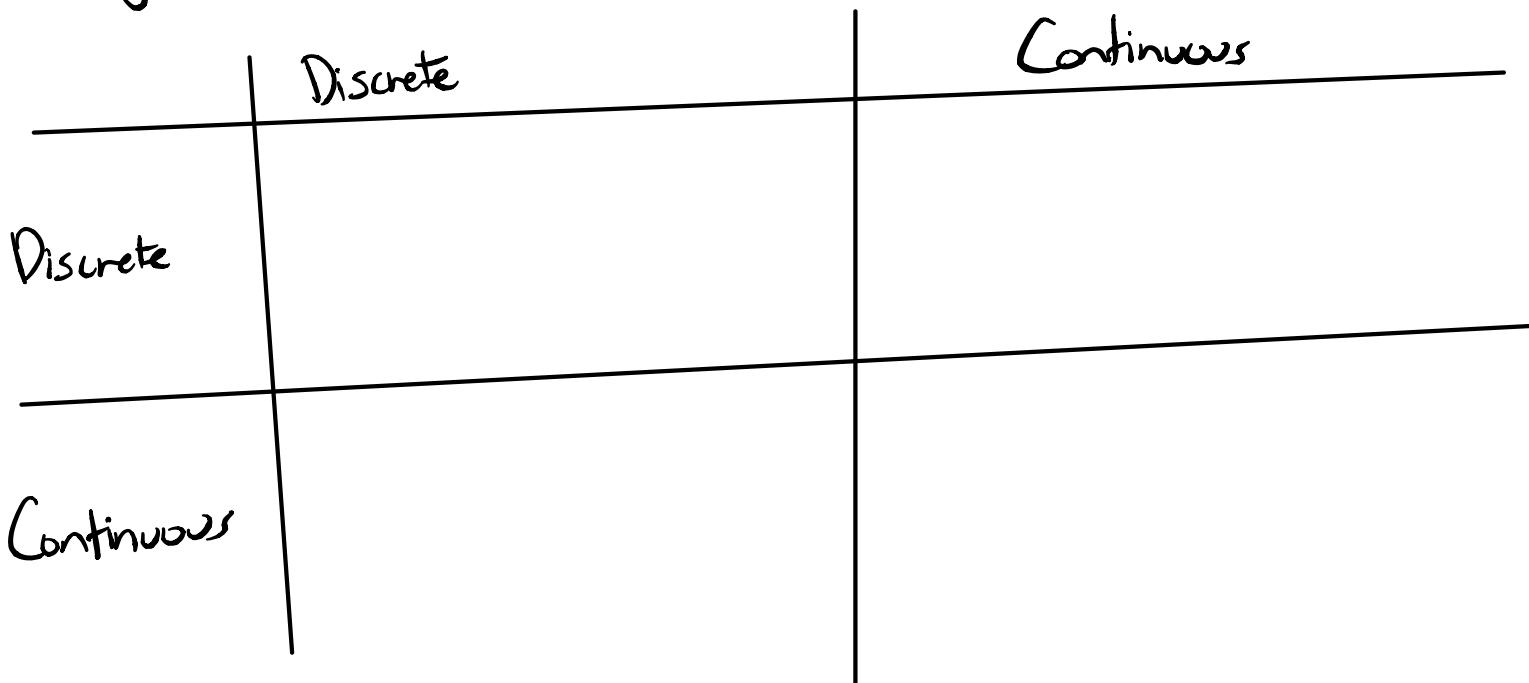


Total Probability Theorem Examples

①

②

Bayes' Rule



Example:
(Discrete / Continuous)