1. Joint PDFs
2. Conditional PDF
3. Independence
4. Marginalization
5. Examples

Joint PDFs

Let X and Y be two continuous random variables.

Then the joint density function $f_{X,Y} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

satisfies :

Joint CDFs

Let X and Y be two random variables.
Then the joint cumulative distribution function
$F_{X,Y} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ satisfies:

Conditional PDFs

Let X and Y be two continuous random variables with joint density function $f_{X,Y}$

For any y with $f(y) > 0$, the conditional distribution of X given $Y = y$ is defined as:

When Y is continuous, even though $P(Y = y) = 0$,
if $f_Y(y) > 0$,

Independence

Let X and Y be two continuous random variables.
X and Y are independent if:

Marginalization

To recover the individual pdfs from the joint pdf:

$f_X(x) =$

$f_Y(y) =$

<u>Example:</u> $X, Y \overset{iid}{\sim} \text{Unif}(0,2)$

What is $f_{X,Y}(x,y)$ for two uniform r.v.s on $[0,2]$?

Uniform Density Over a Disk : Joint

What is $f_{X,Y}(x,y)$ for a uniform density over a disk of radius r centered at the origin?

Uniform Density Over a Disk: Marginals

What is $f_y(y)$ and $f_x(x)$, now that we know the joint over the disk?

Uniform Density over a Disk: Conditional PDFs.

What is $f_{X|Y=y}(x)$?

What about $\mathrm{Cov}(X, Y)$?

2D LOTUS

Let X, Y be two random variable with joint PDF $f_{X,Y}(x,y)$, and let $g(x,y)$ be a real-valued function of x, y. Then,

Expected Distance Between Two Points

Let $X, Y \overset{i.i.d}{\sim} \text{Unif}(0,1)$. What is $\mathbb{E}[|X-Y|]$?

$\mathbb{E}[|X-Y|] =$

Total Probability Theorem

	Discrete Partition	Continuous Partition
Discrete		
Continuous		

Total Probability Theorem Examples

①

②

Bayes' Rule

	Discrete	Continuous
Discrete		
Continuous		

Example:

(Discrete/Continuous)