(1) Markov's Inequality

(2) Chebyshev's Inequality

Reminder:

HW Sessions 3 times a week (@ 39)

Markov's Inequality Intro

Simple bound on the tail of a random variable that only uses the expected value (first moment) and the fact that the random variable is nonnegative.

Markov's Inequality Definition

If X is a nonnegative r.v. with finite mean and $a > 0$, then the probability that X is at least a is at most the expectation of X divided by a

Markov's Inequality: Proof 1

WLOG, let X be a nonnegative continuous R.V.

$\mathbb{E}[X] =$

Markov's Inequality: Proof II

Let I be the indicator r.v. defined as:

$$I = \begin{cases} 1 & \text{if } X \geq a \\ 0 & \text{o.w.} \end{cases}$$

Then

Markov's Inequality: Proof III

$$\mathbb{E}[X] =$$

Example: Markov & Coin Flips

Let $X \sim \text{Geom}\left(\frac{1}{2}\right)$. Use Markov's inequality to bound $P(X > 10)$.

Generalized Markov's Inequality

If X is any random variable with finite mean and $a > 0$, then for an $r > 0$:

Proof left as an exercise.

Chebyshev's Inequality Intro

Often times we can do better than Markov's Inequality if we use more information about the random variable.

For Chebyshev's Inequality we use the first two moments $\mathbb{E}[X]$ and $\mathbb{E}[X^2]$.

Note:

Chebyshev's Inequality: Definition.

doesn't need to be nonnegative.

If X is a random variable with finite mean μ and finite variance, and $c > 0$, then the probability that X is at least c away from μ is at most $\frac{Var(X)}{c^2}$.

Note:

Chebyshev's Inequality: Proof.

X is a r.v.

$\mathbb{E}[X] = \mu$

Let $Y = (X-\mu)^2$,

Example: Chebyshev & Coin Flips

Let $X \sim \text{Geom}(\frac{1}{2})$. Use Chebyshev's Inequality to upper bound $\mathbb{P}(X > 10)$.

Chebyshev Corollary

For any random variable X with finite expectation $\mathbb{E}[X] = \mu$ and finite standard deviation $\sigma = \sqrt{\mathrm{Var}(X)}$:

Example: Chebyshev Corollary

Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Find a bound on the probability that X is 2σ or more away from its mean μ.

Law of Large Numbers: Preview

If we observe a random variable many times and average our observations, the average will converge to the average of the random variable.

Note: