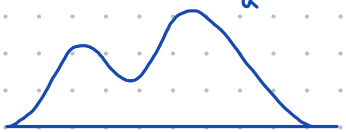
Administrivia

Course evaluations at 26.05% 👁👄👁 If they hit 80%, everyone gets an additional homework drop.

Recap

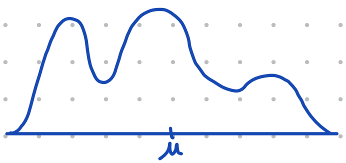
__Thm__ (Markov's Inequality) For any nonnegative random variable X and $a > 0$,

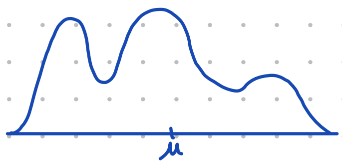
$$\mathbb{P}(X \geq a) \leq \frac{\mathbb{E}[X]}{a}$$



__Thm__ (Chebyshev's Inequality) For any random variable X with expectation μ and variance $\sigma^2 < \infty$,

$$\mathbb{P}(\underbrace{|X - \mu| \geq c}_{\text{X is more than a distance of c from its mean}}) \leq \frac{\mathrm{Var}[X]}{c^2}$$



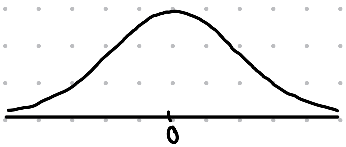


__Note__ Normal Random Variables

For $X \sim \mathrm{Normal}(\mu, \sigma^2)$ and $Z \sim \mathrm{Normal}(0,1)$,

$$\frac{X-\mu}{\sigma} \overset{d}{=}$$

The density of Z is symmetric $(\phi(z) = \phi(-z))$, so



For $X_1, \ldots, X_n$ iid with mean μ and variance σ^2,

Estimation

<u>Def</u> An estimator is a

The bias of an estimator is

$\text{Bias}[X] =$

We say an estimator is unbiased if

<u>Ex</u> Suppose $X_1, \ldots, X_n \overset{iid}{\sim} \text{Bernoulli}(p)$ for unknown parameter p. Construct an unbiased estimator for p.

<u>Ex</u> Suppose $X_1, \ldots, X_n$ are iid with unknown expectation μ and variance σ^2. Construct an unbiased estimator for μ.

<u>Note</u> Generally, for an estimator X of θ, we want that

Chebyshev Confidence Intervals I

Def For $0 < \delta < 1$, a $(1-\delta)$ confidence interval for a fixed parameter θ is

Q We flip a biased coin that flips heads with probability p n times. Let $X_1, \ldots, X_n$ be the results of the flips.
Construct an unbiased estimator for p.

Construct a $(1-\delta)$ confidence interval for p.

Chebyshev Confidence Intervals II

Q We flip a biased coin that flips heads with probability p n times. Let $X_1, \ldots, X_n$ be the results of the flips.
Suppose $n = 1000$ and 120 of the flips are heads. Construct the 95% confidence interval.

Q Suppose $X_1, \ldots, X_n$ are iid with unknown expectation μ and known variance $\sigma^2 = 3$.
Find n such that a 98% confidence interval has error at most 0.01.

Normal Confidence Intervals 1

Q Let $X_1, \dots, X_n$ be iid with expectation μ and variance $\sigma^2 \in (0, \infty)$
What is the approximate distribution of $\bar{X} = \frac{1}{n}\sum X_i$ for large n?

Suppose n is large. Construct a $(1-\delta)$ confidence interval for μ, the population mean.

Note When the sample size is large, the sample standard deviation is a good approximation for σ.

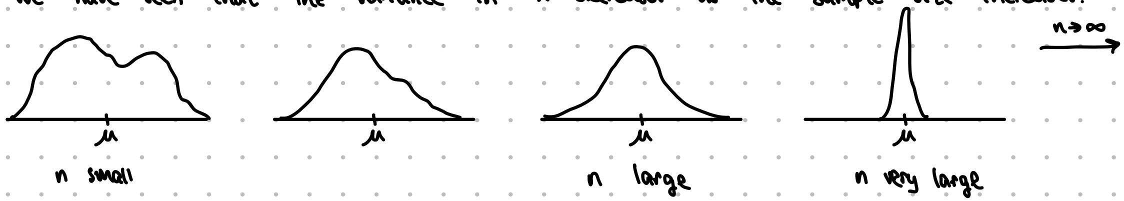
Normal Confidence Intervals 1

Q We flip a biased coin that flips heads with probability p n times. Let $X_1, \dots, X_n$ be the results of the flips.

Suppose $n=1000$ and 120 of the flips are heads. Construct the 95% confidence interval.

Law of Large Numbers

Note We have seen that the variance in $\bar{X}$ decreases as the sample size increases.



Thm (Law of Large Numbers) Let $X_1, \dots, X_n$ be iid with expectation $\mu < \infty$. Let the sample mean

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

For any $\varepsilon > 0$,

$$\mathbb{P}(|\bar{X} - \mu| \leq \varepsilon) \to 1 \quad \text{as} \quad n \to \infty$$

Pf

Ex I flip a biased coin with unknown probability p of heads.
Consider the distribution of $\bar{x}$ for various values of n
[Demo]