

Markov Chains

Def (Markov Property) A process X_0, X_1, \dots obeys the Markov Property if

Ex Consider the process of flipping a coin that flips heads with probability p until we see two consecutive heads.

Claim For X_0, X_1, \dots a Markov chain on S and i_0, i_1, \dots in a sequence of states visited,

Ex Consider the process of flipping a coin that flips heads with probability p until we see two consecutive heads.

$$P(X_0 = H) =$$

$$P(X_0 = T) =$$

$$P(H T H H) =$$

Distribution over Time

Thm Let π_0 be the initial distribution over the state space written as a row vector:

pp

Note To specify a Markov chain, you need

Hitting Time

Q Suppose you repeatedly flip a coin with probability p of heads until you see two consecutive heads. What is the expected number of flips it will take?

Note Let X_0, X_1, \dots be a finite Markov chain on state space S with transition matrix P .

A Before B

Q We repeatedly roll a six-sided die and sum the rolls modulo 3 as we go. What is the chance our sum hits 1 before it hits 2?

Q Consider a sequence of iid trials, each of which results in n mutually exclusive categories outcomes. On each trial, let the chance of category i be $p_i > 0$. What is the chance category i appears before category j ?

Note Let X_0, X_1, \dots be a finite Markov chain on state space S with transition matrix P .

Examples

Q An ant is sitting at the corner of a cube. At each timestep, she traverses an edge uniformly at random. What is the expected time until she reaches the other end of the cube?

